Statistics and Probability Letters 1997 vol.32 N2, pages 167-174

A mean convergence theorem and weak law for arrays of random elements in martingale type p Banach spaces

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Abstract

For weighted sums of the form Sn = ∑kn j=1 anj(Vnj-Cnj) where {anj, 1≤j≤kn < ∞, n≥1} are constants, {Vnj, 1≤j≤kn, n≥1} are random elements in a real separable martingale type p Banach space, and {Cnj, 1≤j≤kn,n≥ 1} are suitable conditional expectations, a mean convergence theorem and a general weak law of large numbers are established. These results take the form ||Sn|| →ℒr 0 and Sn →P 0, respectively. No conditions are imposed on the joint distributions of the {Vnj, 1≤j≤kn, n≥1}. The mean convergence theorem is proved assuming that {||Vnj||r, 1≤j≤kn, n≥ 1} is {|anj|r}-uniformly integrable whereas the weak law is proved under a Cesàro type condition which is weaker than Cesàro uniform integrability. The sharpness of the results is illustrated by an example. The current work extends that of Gut (1992) and Hong and Oh (1995).

Keywords

Array of random elements, Cesàro uniformly integrble array, Convergence in ℒr, Convergence in probability, Real separable martingale type p Banach space, Weak law of large numbers, Weighted sums, {anj}-uniformly integrable array